

2.1)

$$\frac{1}{R} (rR')' = \pm \nu^2$$

$$\frac{\psi''}{\psi} = \mp \nu^2$$

$$\nu=0$$

$$\psi = A_0 + B_0 \varphi$$

$$R = C_0 + D_0 \ln r$$

$$a) \begin{cases} R = ar^{\nu} + br^{-\nu} & , b=0 \text{ (no singular)} \\ \psi = C \cos \nu \varphi + D \sin \nu \varphi \end{cases}$$

$$\phi(r, 0) = V \quad \phi(r, \alpha) = V \Rightarrow \begin{aligned} D_0 &= 0 \\ B_0 &= 0 \\ A_0 C_0 &= V \end{aligned}$$

Otras sol. son compatibles si se anulan en  $\varphi=0, \alpha \quad \forall r \geq 0$

$$\varphi=0 \quad C(ar^{\nu}) = 0 \Rightarrow C=0$$

$$\varphi=\alpha \quad D \sin \nu \alpha ar^{\nu} = 0 \Rightarrow \nu \alpha = n\pi, \quad n=1, 2, \dots$$

$$\Rightarrow \phi(r, \varphi) = V + \sum_{n=1}^{\infty} a_n r^{\frac{n\pi}{\alpha}} \sin(n\pi\varphi/\alpha)$$

b) La sol.  $\nu=0$  es suficiente para las cond. borde.

Las otras son:  $\nu = i\eta, \eta \text{ real}$

$$\begin{cases} R = ar^{i\eta} + br^{-i\eta} = A \cos(\eta \ln r) + B \sin(\eta \ln r) \\ \psi = C \operatorname{ch} \eta \varphi + D \operatorname{sh} \eta \varphi \end{cases}$$

$$\varphi=0: \quad R\psi = R(r)C = 0, \quad C=0$$

$$\varphi=\alpha: \quad R\psi = R(r)D \operatorname{sh} \eta \alpha = 0 \Rightarrow D=0, \quad \text{no son sol.}$$

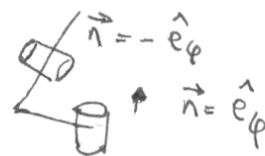
$$2.2) \quad \phi(r, \varphi) = V + a_1 r^{\pi/\alpha} \sin(\pi\varphi/\alpha) \quad (\text{supongamos } a_1 \neq 0)$$

$$E_r = -\frac{\partial \phi}{\partial r} = -\frac{\pi a_1}{r} r^{\pi/\alpha-1} \sin \pi\varphi/\alpha$$

$$E_{\varphi} = -\frac{1}{r} \frac{\partial \phi}{\partial \varphi} = -\frac{\pi a_1}{\alpha} r^{\pi/\alpha-1} \cos \pi\varphi/\alpha$$

$$\text{Gauss: } \sigma(r, 0) = \epsilon_0 E_{\varphi}(r, 0) = -\epsilon_0 \frac{\pi a_1}{\alpha} r^{\pi/\alpha-1}$$

$$\sigma(r, \alpha) = -\epsilon_0 E_{\varphi}(r, \alpha) = \epsilon_0 \frac{\pi a_1}{\alpha} r^{\pi/\alpha-1}$$



$$3) \quad \alpha > \pi \Rightarrow r^{\pi/\alpha-1} \text{ singular}$$

$$\alpha = \frac{3\pi}{2} \quad \sigma \sim r^{-1/3}$$

$$\alpha \rightarrow 2\pi \quad \sigma \sim r^{-1/2}$$

(sin embargo, son integrables y dan carga finita!  
Las campos son no acotados en la arista!)