$\vec{E} = E_0 \frac{s\theta}{r} e^{iu}\left(1+\frac{i}{kr}\right)\hat{\phi}$, $u = kr - \omega t$

$\vec{B} = B_0(\vec{r})\, e^{iu}$

a) $\nabla\times B = -\frac{i\omega}{c^2}\vec{E}$ Ampere

$\nabla\times E = i\omega\vec{B} \rightarrow$ Faraday

$$\nabla\times\vec{E} = \frac{1}{r s\theta}\partial_\theta(s\theta E_\phi)\hat{r} - \frac{1}{r}\partial_r(r E_\phi)\hat{\theta} = 2E_0\frac{c\theta}{r^2}e^{iu}\left(1+\frac{i}{kr}\right)\hat{r} + E_0\frac{s\theta}{r}e^{iu}\left(\frac{1}{r}+\frac{i}{kr^2}-ik\right)\hat{\theta}$$

$$\Rightarrow \vec{B}(\vec{r},t) = \frac{2E_0 c\theta}{\omega r^2}e^{iu}\left(-i+\frac{1}{kr}\right)\hat{r} + \frac{E_0 s\theta}{\omega r}e^{iu}\left(-\frac{i}{r}+\frac{1}{kr^2}-k\right)\hat{\theta}$$

$$\boxed{\vec{B}(\vec{r},t) = \frac{E_0}{\omega r}\left(\frac{2c\theta}{r}\left(su+\frac{cu}{kr}\right)\hat{r} + s\theta\left(\frac{su}{r}+cu\left(\frac{1}{kr^2}-k\right)\right)\hat{\theta}\right)}$$

$\nabla\cdot E = 0?$ $\frac{\partial E_\phi}{\partial\phi} = 0$ ✓

$\nabla\cdot B = 0?$ $\partial_r(r^2 B_r) + \frac{r}{s\theta}\partial_\theta(s\theta B_\theta) = 0$ ✓

$\nabla\times\vec{B} = -\frac{i\omega}{c^2}\vec{E}$ $\quad \nabla\times B = \underbrace{\frac{1}{rs\theta}(-\partial_\phi B_\theta)}_{=0}\hat{r} + \underbrace{\frac{1}{rs\theta}(\partial_\phi B_r)}_{=0}\hat{r} + \frac{1}{r}\left(\partial_r(rB_\theta)-\partial_\theta B_r\right)\hat{\phi}$

$$= \frac{E_0 s\theta}{cr^2}\left(cu + \frac{su}{r}\right)\hat{\phi}\ ✓$$

b) $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $\vec{E} = E_0\frac{s\theta}{r}cu\,\hat{\phi}$ $\quad \boxed{\vec{S} = \frac{E_0^2 s^2\theta}{\mu_0 c r^2}(cu)^2\,\hat{r}}$

$\vec{B} = -\frac{E_0}{\omega r}k s\theta\,\hat{\theta}\,cu$

$I = \vec{S}\cdot r^2\hat{r} = \boxed{\frac{E_0^2 s^2\theta (cu)^2}{\mu_0 c r^2} = I}$

$\overline{(cu)^2} = \frac{1}{2}$ $\quad \boxed{\frac{d\bar{P}}{d\Omega} = \frac{E_0^2\, sen^2\theta}{2\mu_0 c}} \Rightarrow \boxed{\bar{P} = \int\frac{d\bar{P}}{d\Omega}d\Omega = \frac{4E_0^2\pi}{3\mu_0 c}}$